FACTORING TRINOMIALS, ALL MIXED UP

- Before doing this exercise, you may want to study:
  Basic Concepts Involved in Factoring Trinomials
  Factoring Trinomials of the form $x^2 + bx + c$, where $c > 0$
  Factoring Trinomials of the form $x^2 + bx + c$, where $c < 0$

(more mathematical cats)

Here, you will practice factoring trinomials of the form $x^2 + bx + c$,
where b and c are integers.
Notice that the coefficient of the x^2 term is 1.

Recall that the ***integers*** are: $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

As discussed in Basic Concepts Involved in Factoring Trinomials,
you must first find two numbers that add to b and that multiply to c, since then:

$$x^2 + bx + c \;=\; x^2 + (\overbrace{f+g}^{=b})x + \overbrace{fg}^{=c} \;=\; (x+f)(x+g)$$

As discussed in Factoring Trinomials of the form $x^2 + bx + c$, where $c > 0$:
if c is positive, then both numbers will be positive, or both numbers will be negative.
When you add numbers that have the same sign, then in your head you do an addition problem.

As discussed in Factoring Trinomials of the form $x^2 + bx + c$, where $c < 0$:
if c is negative, then one number will be positive, and the other will be negative.
When you add numbers that have different signs, then in your head you do a subtraction problem.

THE 'PANS' MEMORY DEVICE

When you're trying to find the ***two numbers that work***,
you always want to do the mental arithmetic with only positive numbers.
It's much easier this way.
Here are the steps:

- Check that the coefficient of the square term is 1.
- Look at the constant term (c).
  This is the number that tells you whether, in your head, you'll think of adding or subtracting.
  Is it Positive? Then, you'll be Adding.
  Is it Negative? Then, you'll be Subtracting.
  This is the PANS memory device!
  Positive Addition Negative Subtraction

- Throw away all the minus signs (if any)
  and find two numbers that multiply to $|c|$ and add/subtract (as appropriate) to $|b|$.
  Lots of examples below.
- Adjust the sign(s) of your two numbers, as needed.
  (Read the details about this in the earlier web exercises.)
- Use your two numbers to factor the trinomial, as illustrated in the examples below.
- Be sure to check your answer using FOIL.

EXAMPLES:

Factor: $x^2 + 5x + 6$
Solution:

- Is the coefficient of the x^2 term equal to 1? Check!
- Look at the constant term, 6.
  It's positive.
  PANS—**P**ositive, **A**dd.
- There aren't any minus signs to throw away.
  Find two numbers that multiply to 6 and add to 5.
  Got them? 2 and 3.
- That 5 in the middle is positive.
  So, both numbers will be positive.
  The two numbers are 2 and 3.
- $x^2 + 5x + 6 \;=\; x^2 + (\overbrace{2+3}^{=5})x + \overbrace{2 \cdot 3}^{=6} \;=\; (x+2)(x+3)$

Factor: $x^2 + 5x - 6$
Solution:

- Is the coefficient of the x^2 term equal to 1? Check!
- Look at the constant term, -6.
  It's negative.
  PA**NS**—**N**egative, **S**ubtract.
- Throw away all minus signs.
  Find two numbers that multiply to 6 and subtract to give 5.
  Got them? 6 and 1.
- That 5 in the middle is positive.
  So, the bigger number will be positive.
  The two numbers are 6 and -1.
- $x^2 + 5x - 6 \;=\; x^2 + (\overbrace{6+(-1)}^{=5})x + \overbrace{6 \cdot (-1)}^{=-6} \;=\; (x+6)(x-1)$

Factor: $x^2 - 5x + 6$
Solution:

- Is the coefficient of the x^2 term equal to 1? Check!
- Look at the constant term, 6.
  It's positive.
  PANS—**P**ositive, **A**dd.
- Throw away all minus signs.
  Find two numbers that multiply to 6 and add to 5.
  Got them? 2 and 3.
- That -5 in the middle is negative.
  So, both numbers will be negative.
  The two numbers are -2 and -3.
- $x^2 - 5x + 6 \;=\; x^2 + (\overbrace{(-2) + (-3)}^{=-5})x + \overbrace{(-2) \cdot (-3)}^{=6} \;=\; (x-2)(x-3)$

Factor: $x^2 - 5x - 6$
Solution:

- Is the coefficient of the x^2 term equal to 1? Check!
- Look at the constant term, -6.
  It's negative.
  PA**NS**—**N**egative, **S**ubtract.
- Throw away all minus signs.
  Find two numbers that multiply to 6 and subtract to give 5.
  Got them? 6 and 1.
- That -5 in the middle is negative.
  So, the bigger number will be negative.
  The two numbers are -6 and 1.
- $x^2 - 5x - 6 \;=\; x^2 + (\overbrace{(-6) + 1}^{=-5})x + \overbrace{(-6) \cdot 1}^{=-6} \;=\; (x-6)(x+1)$

Factor: $x^2 + 3x - 7$
Solution:

- Is the coefficient of the x^2 term equal to 1? Check!
- Look at the constant term, -7.
  It's negative.
  PA**NS**—**N**egative, **S**ubtract.
- Throw away all minus signs.
  Find two numbers that multiply to 7 and subtract to give 3.
  There aren't any.
  $x^2 + 3x - 7$ is not factorable over the integers.